

## PERFORMANCE ANALYSIS OF AREA SPECTRAL EFFICIENCY OF MASSIVE MIMO CELLULAR SYSTEM

J.Roscia Jeya Shiney<sup>1</sup> and K.Nivetha<sup>2</sup>

<sup>1</sup>Dept. of Electronics and Communication Eng, Mepco Schlenk Engineering College, Sivakasi, TamilNadu, rosciashiney@yahoo.co.in

<sup>2</sup>M.E Communication Engineering, Mepco Schlenk Engineering College, Sivakasi, TamilNadu jnivekan@gmail.com

### ABSTRACT

In the wireless world, 5G is one of the emerging technologies. Massive Multiple Input Multiple Output (MIMO) technology used in 5G wireless communication where high frequency signals are used. Massive MIMO is an antenna array system using massive amount of antennas. Extra antennas help by focusing the transmission and reception of signal energy into ever-smaller regions of space. Though many techniques are used for 5G, massive MIMO gives good performance measures. Spectral Efficiency has been an important performance measure for mobile cellular systems. This work concentrates on area spectral efficiency (ASE) for uplink multi-cell multi-user massive MIMO systems by using a uniformly distributed user location model. This work presents the analysis of area spectral efficiency and some system parameters such as the number of base station antennas, the number of users and pilot-to-data power ratio of the massive MIMO cellular system. The area spectral efficiency is achieved for different path loss models and by considering different SNR values.

**Key Words:** Area Spectral Efficiency, Massive MIMO, Cellular System, 5G wireless communications, Multi-cell multi-user

### I. INTRODUCTION

The fifth generation (5G) mobile communication, spectral efficiency must be increased compared to the fourth generation (4G) mobile communication. It is a challenging task for new technologies. To improve spectral efficiency (SE) for 5G system, massive MIMO technique has been proposed.

In massive MIMO cellular systems, antennas are deployed with hundreds or thousands of active element at base station (BS). The number of antennas  $M$  should be more than scheduled users  $K$  because users' channel is to be near-orthogonal when  $M/K > 10$  [2]. In massive MIMO, the multi-user interference can be reduced to zero when number of antennas at the base station is infinity [3-5].

Spectral Efficiency (SE) has been an important performance measure for mobile cellular systems. The cell-average spectral efficiency specifies the average spectral efficiency over all the active mobile users present in a cellular system [6-9]. Area Spectral Efficiency is one of the metric of the cell-average spectral efficiency [10].

The unit of ASE is bits/s/Hz/km<sup>2</sup>. In this paper, ASE is used to quantify the SE performance of massive MIMO cellular systems.

### II. MASSIVE MIMO SYSTEM

A closed-form approximation of area spectral efficiency is obtained for massive MIMO cellular systems under uniformly distributed user location model. The optimal number of users and optimal pilot-to-data power ratio is obtained for ASE maximization [1].

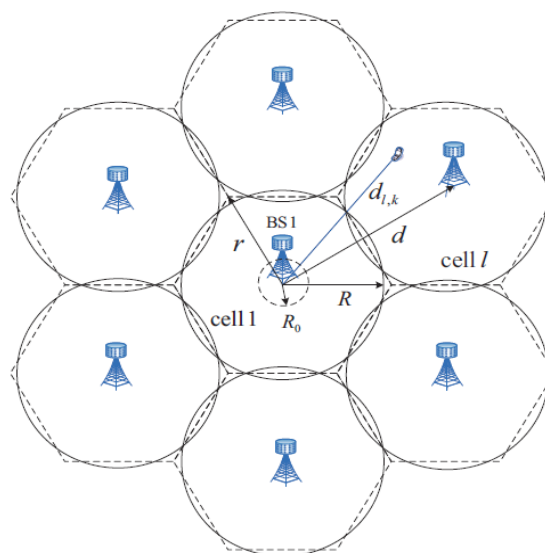


Fig.1. Massive MIMO system

The multi-cell multi-user MIMO system is consist of  $L$  hexagonal cells with radius  $r$ . Each base station has  $M$  antennas and each cell has  $K$  users with single antenna. Cell1 is the reference cell and the other  $L-1$  cells are the interfering cells. The distance between base station 1 and adjacent base station is denoted as  $d$ .  $R_0$  is the closest distance from the mobile to base station[1].

#### A. Channel Model

The users are assumed to be independently and uniformly distributed in all of the cells. The uplink channel between the  $k$ -th user of the  $l$ -th cell and base station1 is denoted as,

$$g_{l,k} = \sqrt{\lambda_{l,k}} h_{l,k} \quad (1)$$

where  $g_{l,k}$  denotes the large scale fading. The large scale fading plays an important role in massive MIMO systems[9]. In this paper, the large scale fading as,

$$\lambda_{l,k} = cd_{l,k}^{-\alpha} \quad (2)$$

where  $d_{l,k}$  is the distance between the k-th user of the l-th cell and base station1,  $\alpha$  is the path loss exponent typically between 3.0 and 5.0 and  $c$  is the median of mean path gain at a reference distance  $d_{l,k}=1\text{km}$ , which is related to the antenna pole height in meters, the carrier frequency in MHz and the propagation environment.

### III. ASE ANALYSIS OF MASSIVE MIMO

The area spectral efficiency of massive MIMO system is studied for different path loss models. First the asymptotic sum-rate of massive MIMO is analyzed when number of antennas becomes infinity. Then under uniformly distributed user location model, a closed form approximation of area spectral efficiency is obtained. Finally, the optimal number of users is studied for maximum area spectral efficiency.

#### A. Sum-Rate Analysis

The sum-rate capacity is analyzed with joint MMSE receiver [8]. The sum-rate capacity gives the maximum aggregation of all users data. For massive MIMO, the sum-rate of the system is decoupled into K parallel single-user multi-cell systems. The sum-rate capacity is given by,

$$\tilde{C} \approx K \Sigma \log \left( 1 + \frac{d_1^{-2\alpha}}{\sum_{l=2}^L d_l^{-2\alpha} + \frac{\varepsilon + \gamma/P_D}{M} \left( \sum_{l=1}^L d_l^{-\alpha} + \frac{\gamma}{KP} \right)} \right) \quad (3)$$

Where,

$$\varepsilon = \sum_{k=1}^K \sum_{l=1}^L d_{l,k}^{-\alpha} - \sum_{k=1}^K \left( \sum_{l=1}^L d_{l,k}^{-2\alpha} \right) \left( \sum_{l=1}^L d_{l,k}^{-\alpha} + \frac{\gamma}{KP} \right)^{-1}$$

$$\gamma = \frac{\sigma_n^2}{c}$$

To evaluate the system-level performance, reference SNR (RSNR) as,

$$RSNR = \frac{P_D}{\gamma} \quad (4)$$

The equation (3) shows the system performance in the limit of infinitely many base station antennas and users and the expectation is taken with static characteristics of uniform located users.

#### B. ASE For Massive MIMO

The channel remains constant over a coherence interval of T symbols[1]. During a coherence interval, each user first transmits K pilot symbols and then transmits T-K data symbols in the remainder of the interval. The ASE of reference cell is defined as,

$$C_{ASE} = \frac{T-K}{ST} \tilde{C} \quad (5)$$

Where,

$$S = \pi(R^2 - R_0^2)$$

S is the area of the cell excluding the tiny central part of the cell.

When the scale of wireless communication system is large, that is L is large, using Lemma 1 in [20] and given by,

$$\tilde{C} = K \log \left[ 1 + \frac{\phi_2(d_1)}{\phi_2(d_1) + \frac{\varepsilon + \gamma/P_D}{M} \left( \phi_1(d) + \frac{\gamma}{KP} \right)} \right] \quad (6)$$

Where  $d=[d_1, d_2, \dots, d_L]^T$ ,  $d_{[1]}=[d_2, \dots, d_L]^T$

In general,  $\phi_n(z)$  is defined as,

$$\phi_n(z) = \epsilon \left( \sum_i^m z^{-n\alpha} \right) \quad (7)$$

By simplifying,

$$C_{ASE} = \frac{T-K}{ST} K \times \log \left( 1 + \frac{\phi_2(d_1)}{\frac{\phi_1(d)}{MP_D} \left( \frac{1}{\beta} + 1 \right) + \phi_2(d_{[1]}) + \frac{K}{M} \left( \phi_1^2(d) - \phi_2(d) + \frac{\gamma}{KP} \right)} \right) \quad (8)$$

Where  $\beta$  is the pilot-to-data power ratio as

$$\beta = \frac{P}{P_D}$$

$$\phi_2(d_{[1]}) = \exp(-\alpha\epsilon(D_{min})) + (L-2)\exp(-\alpha\epsilon(D_{max}))$$

A closed-form approximation of ASE which is associated with basic system parameters such as the cell radius, number of antennas at BS and the number of users.

#### C. Optimal Number Of Users

The area spectral efficiency depends on the number of users. For very large M, the sum-rate is linearly increasing with the number of users K. Considering the training loss, the ASE does not always increase with K[1]. Given the coherent time T there is a maximum number of users that the system can support. At low SNR, ASE is approximated as,

$$C_{ASE} = \frac{T-K}{ST \ln 2} K \left( \frac{\phi_2(d_1)}{\Delta_1 + \Delta_2 K + \Delta_3 / K} \right) \quad (9)$$

Where

$$\Delta_1 = \frac{\phi_1(d)\gamma(1 + \beta)}{MP_D} + \phi_2(d_1)$$

$$\Delta_2 = \frac{1}{M}(\phi_1^2(d) - \phi_2(d))$$

$$\Delta_3 = \frac{\gamma^2}{\beta MP_D^2}$$

After some mathematical manipulations, the optimal number of users is given as,

$$K = \sqrt[3]{\left(-\frac{\Omega_1}{2}\right) + \sqrt{\left(\frac{\Omega_1^2}{4}\right) + \left(\frac{\Omega_2^3}{27}\right)}} + \sqrt[3]{\left(-\frac{\Omega_1}{2}\right) + \sqrt{\left(-\frac{\Omega_1^2}{4} - \frac{\Omega_2^3}{27}\right)}}$$

where, (10)

$$\Omega_1 = \frac{3\Delta_3 - T\Delta_1}{\Delta_2} - \frac{4\Delta_1^2}{3\Delta_2^2}$$

$$\Omega_2 = -\frac{2T\Delta_3}{\Delta_2} + \frac{16\Delta_1^3}{27\Delta_2^3} - \frac{6\Delta_1\Delta_3 - 2T\Delta_1^2}{3\Delta_2^2}$$

#### IV. PATH LOSS MODEL

The path loss model considered in this paper is modified COST231 Hata model and distance based path loss model. The modified COST 231 Hata model is a widely used radio propagation model. The modified COST 231 Hata model covers a wider range of frequencies. The modified COST231 Hata urban propagation model is studied[14]. The modified COST231 Hata model have path loss exponent and median of mean path gain value is analyzed [1].The distance based path loss model is studied and the value of path loss exponent and median of mean path gain is obtained..

#### V. RESULTS AND DISCUSSION

The results of area spectral efficiency and optimal pilot-to-data power ratio are analyzed.

##### A. Distance

The distance between base station1 and the users is calculated for uniformly distributed user location model. The distance of the user is distributed equally in a cell for uniformly distributed user location model.

Table 1 Distance in kilometer

d <sub>1,1</sub> = 1.090	d <sub>1,11</sub> =1.3625	d <sub>1,21</sub> =1.6623	d <sub>1,31</sub> =1.9348
d <sub>1,2</sub> =1.1173	d <sub>1,12</sub> =1.3898	d <sub>1,22</sub> =1.6895	d <sub>1,32</sub> =1.9620
d <sub>1,3</sub> =1.1445	d <sub>1,13</sub> =1.4170	d <sub>1,23</sub> =1.7168	d <sub>1,33</sub> =1.98620
d <sub>1,4</sub> =1.1718	d <sub>1,14</sub> =1.4443	d <sub>1,24</sub> =1.740	d <sub>1,34</sub> =1.9893
d <sub>1,5</sub> =1.1990	d <sub>1,15</sub> =1.4715	d <sub>1,25</sub> =1.7713	d <sub>1,35</sub> =2.0165
d <sub>1,6</sub> =1.2263	d <sub>1,16</sub> =1.4988	d <sub>1,26</sub> =1.7985	d <sub>1,36</sub> =2.0438
d <sub>1,7</sub> =1.2535	d <sub>1,17</sub> =1.5533	d <sub>1,27</sub> =1.8258	d <sub>1,37</sub> =2.0710
d <sub>1,8</sub> =1.2808	d <sub>1,18</sub> =1.5805	d <sub>1,28</sub> =1.8530	d <sub>1,38</sub> =2.0983
d <sub>1,9</sub> =1.3080	d <sub>1,19</sub> =1.6078	d <sub>1,29</sub> =1.8803	d <sub>1,39</sub> =2.1255
d <sub>1,10</sub> =1.3353	d <sub>1,20</sub> =1.6350	d <sub>1,30</sub> =1.9075	d <sub>1,40</sub> =2.1528

##### B. Sum-Rate Capacity

The sum-rate capacity increases linearly for number of users. For modified COST231 Hata model the sum-rate capacity obtained is 87 bits per channel use. Table 2 Table for path loss exponent and sum-rate capacity

The table 5.1 shows the distance between K users of the l-th cell.

Path Loss Exponent	Sum-rate Capacity
3.5	87 bits per channel use
3.7	92 bits per channel use

The sum rate capacity is different for different path loss models. The sum rate capacity increases for increasing path loss exponent.

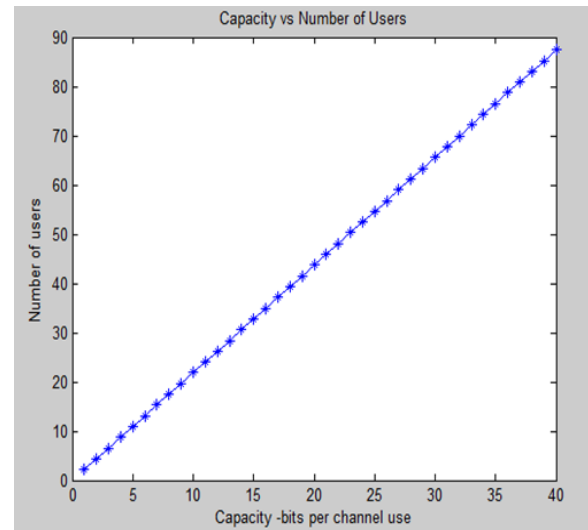


Fig.3 Sum-Rate Capacity for modified COST231 Hata model

The sum-rate capacity for distance based path loss model is 92 bits per channel use. The sum-rate capacity is high for distance based path loss model. The sum-rate capacity is depends on path loss exponent value. Higher the path loss exponent, intercell interference is reduced.

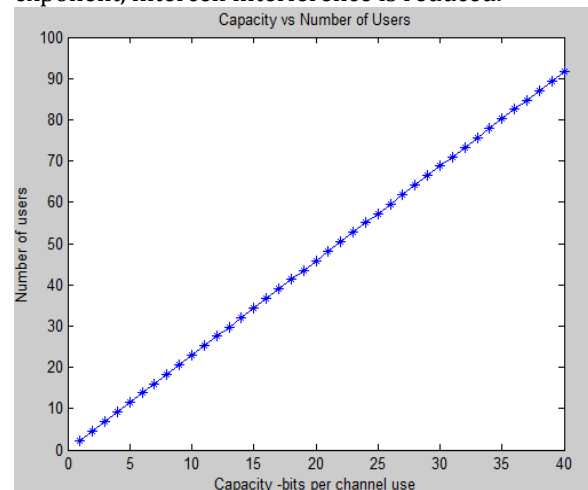


Fig.4 Sum-Rate Capacity for distance based path loss model.

**C. Area Spectral Efficiency**

Area spectral efficiency for a reference cell is obtained by considering pilot-to-data power ratio (PDPR). A closed form PDPR maximizes the spectral efficiency.

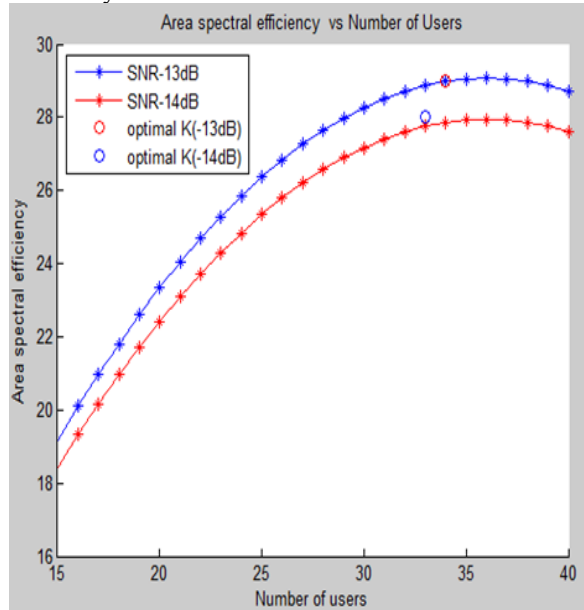


Fig.4 ASE for modified COST231 Hata model

The area spectral efficiency of massive MIMO cellular systems increases for number of users. When number of users is large, ASE goes into saturation area. So, optimization is done for number of users. For SNR -13dB, ASE is 28.7 bits/s/km<sup>2</sup> and optimal number of users is 34. For SNR -14dB, ASE is 27.5 bits/s/km<sup>2</sup> and optimal number of users is 33.

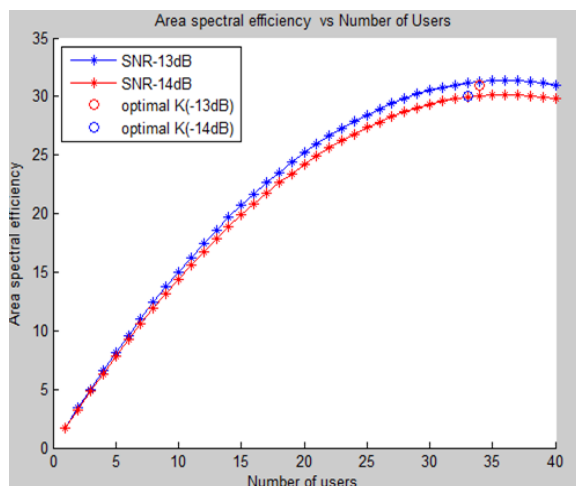


Fig.5 ASE for distance based path loss model

For SNR -13dB, ASE is 31.5 bits/s/km<sup>2</sup> and optimal number of users is 34. For SNR -14dB, ASE is 29.8 bits/s/km<sup>2</sup> and optimal number of users is 33. The area spectral efficiency is increased for distance based path loss model.

Table.4 Comparison for different path loss models

Parameter	Modified COST231 Model	Distance based path loss model
$\alpha$	3.5	3.7
Sum-rate capacity	87 bits per channel use	92 bits per channel use
Area spectral efficiency	28.7 bits/s/km <sup>2</sup>	31.5 bits/s/km <sup>2</sup>

**VI CONCLUSION**

A closed-form approximation of ASE for multi-cell multi-user massive MIMO was analyzed. Some parameters for system design, such as the number of users, number of antennas and the PDPR, were optimized based on ASE maximization criterion. For ASE maximization the system always has better performance with the increasing number of antennas. More antennas can satisfy a higher ASE requirement, however, the growing number of users can not always guarantee higher ASE. Hence, optimal number of users is not gradually increasing with the demand of ASE. ASE is compared for distance based path loss model and modified COST231 Hata model. ASE is maximum for distance based path loss model when compared to modified COST231 Hata model.

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